2022

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer in their own words as far as practicable.
Symbols used have their usual meaning.

GROUP – A

1. Answer all questions and fill in blanks as required. [1 × 12

(a) For the circle $|z| = 1$, the inverse of the point z is _________

(b) If the amplitude of the complex number z be θ, then the amplitude of (iz) is _________.

(c) The polar form of a complex number $-5 + 5i$ is _________.

(d) The function $f(z) = e^{2z}$ is continuous and differentiable everywhere. Write TRUE or FALSE.

(e) The radius of the circle of the convergence of the power series is called _________.

(f) Write Cauchy Riemann equations in polar form.

(g) The radius of convergence of the series $\sum_{n=1}^{\infty} n^n z^n$ is ________.

(h) If C is the closed curve with z = a inside C, then $\int_C \frac{dz}{z-a} =$ ________.

(i) A contour is a continuous chain of a finite number of rectangular arcs. Write TRUE or FALSE.

(j) Residue of $\frac{1}{z^3 - z^5}$ at $z = \pm 1$ is ________.

(k) Does $\lim_{z \to z_0} \left(\frac{\bar{z}}{z}\right)$ exist ?

(l) The radius of the circle given by $\left|\frac{z-i}{z+i}\right| = 5$ is ________.

GROUP – B

2. Answer any eight of the following questions . [2 × 8

(a) Show that $|z|^2 = z.\bar{z}$

(b) Define Residue of f(z) at infinity.

(c) If $v = 4x^3y - 4xy^3$, then find value of analytic function $f(z) = 4 + iv$.

(d) Find radius of convergence of power series $\sum \frac{n!}{n^2} z^n$.

(e) If C represents a square bounded by $x = \pm a$, $y = \pm a$, then find value of $\int_C \frac{dz}{z}$.

(f) State Cauchy's integral formula.

(g) Evaluate $\int_C z^2\, dz$ where C is boundary of the triangle with vertices $0, 1 + i, -1 + i$ clockwise.

(h) State Morera's theorem.

(i) Define zero of an analytic function.

(j) Write the number of zeros of the function

$$f(z) = \sin\left(\frac{1}{z}\right).$$

GROUP – C

3. Answer <u>any eight</u> of the following questions. [3 × 8

(a) Prove that $| z_1 - z_2 | \geq | z_1 | - | z_2 |$.

(b) Discuss the continuity of the function $f(z) = \frac{\bar{z}}{z}$ at $z = 0$..

(c) Evaluate $\int_{0}^{1+i}\left(x^2+iy\right)dz$ along the path y = x.

(d) If f(z) has a pole at z = a, then prove that $\lim_{z\to a} f(z)=\infty$.

(e) State properties of complex line integrals.

(f) Show that f(z) = $\overline{z}$ is not analytic everywhere.

(g) If C is the circle | z – a | = r, then prove that $\frac{1}{2\pi i}\int_{C}\frac{dz}{z-a}=1$.

(h) Find Taylor's series expansion of the function $f(z)=\frac{z}{z^4+9}$ around z = 0.

(i) Find value of $\oint_{|z=3|}\frac{e^z}{z-2}dz$.

(j) Show that $\frac{1}{z^4+2z^2+1}$ has two double poles.

GROUP – D

*Answer **any four** questions.*

4. Find region of the z-plane for which | z – 1 | + | z + 1 | ≤ 3. [7

5. Prove that the function $e^x(\cos y + i \sin y)$ is holomorphic and find its derivative. [7

6. Show that the domain of the convergence of the series $\sum\left(\frac{iz-1}{2+i}\right)^n$ is given by $|z+i|<\sqrt{5}$. [7

7. Evaluate $\int_c \frac{e^{2z}dz}{(z+1)^4}$ where C is $|z|=3$. [7

8. Expand $\frac{2z+3}{z+1}$ in powers of $z-1$. Find radius of circle of convergence. [7

9. Find order of poles and values of residues of the function $f(z)=\frac{z+3}{z^2-2z}$. [7

10. Prove that $\cot z=\frac{1}{z}+2z\sum_{n=1}^{\infty}\frac{1}{z^2-n^2\pi^2}$. [7

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GROUP – A

1. Answer all questions and fill in blanks as required. [1 × 12

(a) The group of automorphism of a finite cyclic group is abelian. State True or False.

(b) Every Inner automorphism is an automorphism. State True or False.

(c) Every subgroup of finite cyclic group is a characteristic subgroup. State True or False.

(d) $\text{Inn}(G) =$ ________ iff G is abelian.

(e) The product of two commutators is also a commutator. State True or False.

(f) $O(\text{Inn}(U(n))) =$ __________.

(g) For external direct product, H and K can be any groups. State True or False.

(h) Commutator of x and n is ________, for $n \in G$.

(i) Every G-set is also a group. State True or False.

(j) $Cl(a) =$ ________ iff $a \in Z(G)$

(k) If $n \geq 6$, then every non-trivial conjugacy class in S_n and A_n has atleast __________ elements.

(l) The homomorphism from a simple group is either trivial or one-one. State True or False.

GROUP – B

2. Answer <u>any eight</u> of the following questions. [2 × 8

(a) Find order of $Z_3 \times Z_4$.

(b) Is order of all conjugate elements are same. Justify your answer.

(c) Let $G = S_3$. Write conjugate class of (1 2 3).

(d) Let G be a group for $a, b \in G$, then prove that

$$[b, a] = [a, b]^{-1}.$$

(e) If $f : D_4 \rightarrow D_4$, then how many inner-automorphisms of f are there ?

(f) Let G be a group of order 36. Then show that G has normal subgroup of order 3 or 9.

(g) State Sylow 2nd theorem.

(h) State Index theorem.

(i) Define self conjugate elements of a group.

(j) State Sylow's 3rd theorem.

GROUP – C

3. Answer any eight of the following questions. [3 × 8

(a) Let G be an abelian Group and $f : G \to G$ such that $f(x) = x^{-1}$. Show that f is an automorphism.

(b) If $O(\text{Aut}(g)) > 1$, then show that $O(G) > 2$.

(c) Let $G = \{e, a, a^2, a^3\}$ be a cyclic group of order 4. Then show that $H = \{e, a^2\}$ is a characteristic subgroup of G.

(d) Show that any two conjugate subgroups of a group G are isomorphic.

(e) Describe Sylow p-subgroup of S_3.

(f) Find the groups of order 99.

(g) Define stabilizer and kernel of group actions.

(h) Show that goups of order p^2 are abelian.

(i) Prove that D_4 cannot be expressed as an internal direct product of its two proper subgroups.

(j) Let G' be a commutator subgroup of a group G. Then prove that G is abelian iff $G' = \{e\}$, where e is the identity element of G.

GROUP – D

*Answer **any four** questions.*

4. Prove that the set Inn(G) of all inner automorphisms of a group G is a normal subgroup of the group Aut(G) of its automorphism. [7

5. Prove that if a group G is the internal direct product of a finite number of subgroups H_1, H_2, ..., H_n, then G is isomorphic to the external direct product of H_1, H_2, ..., H_n. [7

6. If G is a finite group of order n and p is smallest prime dividing n, then any subgroup of Index p is normal. [7

7. Every group G of order p^2, where p is prime is isomorphic to Z_{p^2} or $Z_p \oplus Z_p$. [7

8. State that prove Generalized Cayley theorem. [7

9. Let the group D_8 acts on a set of cosets of subgroup H = <S> by left multiplication. Find the permutation representation associated to this action. [7

10. Determine all abelian groups upto isomorphism of order 16. [7

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GROUP – A

1. Answer all questions. [1 × 12

(a) Define curvature.

(b) Write the equation of normal plane.

(c) Find value of [t n b].

(d) Define evolute.

(e) Write the equation of tangent plane to the surface $F(x, y, z) = 0$.

(f) Write Rodrigue's formula.

(g) Define envelope.

(h) Write the relation among H, E, F, G.

(i) Write the condition for a surface be developable.

(j) Write the condition for minimal surface.

(k) Find value of $t' . b'$.

(l) What is the value of $LN - M^2$?

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) Define skew curvature.

(b) Define Rectifying plane.

(c) Write the equation of edge of regression.

(d) Define asymptotic lines.

(e) State Beltrani and Enneper theorem.

(f) Define conjugate direction at a point on a surface.

(g) What are conditions for parametric curves to be asymptotic lines ?

(h) Write the condition for asymptotic lines to be orthogonal.

(i) Define Geodesic.

(j) Write canonical geodesic equations.

GROUP – C

3. Answer any eight of the following questions within 75 words each.

[3 × 8

(a) Prove that torsion on a plane curve vanishes.

(b) Prove that $[r' \, r'' \, r'''] = k^2\tau$.

(c) Prove the relation :

$$\frac{d}{ds}\left[\sigma \frac{d}{ds}\left(\sigma \frac{d^2r}{ds^2}\right)\right] + \frac{d}{ds}\left(\frac{\sigma}{\rho} \frac{dr}{ds}\right) + \frac{\rho}{\sigma} \frac{d^2r}{ds^2} = 0 \cdot$$

(d) Find the equation to the tangent surface to the curve

$$r = \left(u, \; u^2, \; u^3\right).$$

(e) The distance between corresponding points of two involutes is constant. Prove it.

(f) The tangents to Bertand curves at the corresponding points are inclined at a constant angle. Prove it.

(g) Find the equation of tangent plane to the surface

$z = x^2 + y^2$ at $(1, -1, 2)$.

(h) Find E, F, G, H for the paraboloid $x = u, y = v, \; z = u^2 - v^2$.

(i) Find principal radii at the origin of the paraboloid

$2z = 5x^2 + 4xy + 2y^2$.

(j) Prove that the surface $xy = (z - i)^2$ is developable.

GROUP – D

Answer ***any four*** *questions.*

4. State and prove Serret Frenet formula. [7

5. Prove that each characteristic touches the edge of regression. [7

6. Calculate the fundamental magnitudes $x = u \cos \phi$; $y = u \sin \phi$, $z = f(u)$. [7

7. State and prove Euler's theorem on normal curvature. [7

8. For the curve $x = a(3u - u^3)$, $y = 3au^2$, $z = a(3u + u^3)$, prove that

$$k = \tau = \frac{1}{3a\left(1+u^2\right)^2}.$$ [7

9. Calculate the first and second curvature of the helicoid [7

$$x = u \cos v,\ y = u \sin v,\ z = f(u) + cv$$

10. The geodesics at right angles have their torsion equal in magnitude but opposite in sign. Prove it. [7